RATE DISTORTION ANALYSIS OF LEAKY PREDICTION LAYERED VIDEO CODING USING QUANTIZATION NOISE MODELING

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ABSTRACT

Unlike conventional layered scalable video coding, leaky prediction layered video coding (LPLC) introduces a leaky factor α, which takes on values in the range between 0 and 1, to partially include the enhancement layer in the motion compensation loop, hence obtaining a trade-off between coding efficiency and error resilience performance. In this paper, we use quantization noise modeling to theoretically analyze the rate distortion performance of LPLC. An alternative block diagram of LPLC is first developed, which significantly simplifies the theoretical analysis. Closed form expressions, as a function of the leaky factor, are derived for two scenarios, where drift error occurs in the enhancement layer and no drift occurs within the motion compensation loop. Theoretical results are evaluated with respect to the leaky factor, showing that a leaky factor of 0.4-0.6 is a good choice in terms of the overall rate distortion performance of LPLC.

1. INTRODUCTION

Due to the potential incompleteness or loss of the enhancement layer, traditional layered coding usually does not incorporate the enhancement layer in motion compensated prediction (MCP). This is done to prevent drift at the decoder. Leaky prediction layered video coding (LPLC) allows part of the enhancement layer to be used in the MCP to improve the coding efficiency while maintaining graceful error resilience performance [1]. The enhancement layer is allowed to "leak" or be partially used by the MCP. The amount of leak is controlled by a leaky factor between 0 and 1. Theoretical analysis of MCP based video coding which was derived from rate distortion theory was presented in [2]. In [3], rate distortion analysis of non-scalable video coding using leaky prediction was discussed by modeling the video signal as a first-order Markov model. The rate distortion analysis of traditional layered video coding is described in [4] and [5], and the rate distortion analysis of LPLC is addressed in [6]. In this paper, we present a different approach to analyze the rate distortion performance of LPLC by using quantization noise modeling proposed in [7].

2. LEAKY PREDICTION LAYERED VIDEO CODING

As shown in Fig. 1, unlike conventional layered coding structure, LPLC introduces a second MCP step in the enhancement
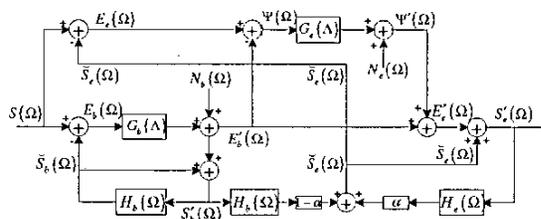
Fig. 1. Block diagram of LPLC

layer using the same motion vectors as the base layer, and buffers $\alpha(s'_e(t) - s'_b(t)) + s'_b(t)$ as the reference for the encoding of the video frame at time $t + \Delta t$. Equivalently, a linear combination of the two reconstructed frames $\{s'_e\}$ and $\{s'_b\}$, namely $\alpha s'_e(t) + (1 - \alpha)s'_b(t)$, is utilized as the reference. We define the mismatch signal, $\{\psi\}$, as the difference between the MCP error signal in the enhancement layer, $\{e_e\}$, and the encoded MCP error signal in the base layer, $\{e'_b\}$. $\{\psi\}$ is coded and carried by the enhancement layer.

In Fig. 1, we use the optimum forward channel to model the encoding process of a 2D image signal [8]. Using this model, a 2D image is encoded at the rate distortion bound if the 2D image is assumed to be a Gaussian stationary random signal. $H_b(\Omega)$[1] is a 3D filter combining both motion compensation operation and spatial filtering for the MCP step in the base layer, while $H_e(\Omega)$ is for the enhancement layer ($\Omega \triangleq (\omega_x, \omega_y, \omega_t) \triangleq (\ ,\omega_t)$) [2]. It is shown in [2] that the optimal spatial filter can be approximated by $F_{opt}(\ ) \approx P^*(\ )$, where $P(\ )$ denotes the characteristic function of the estimated motion vector error $(\Delta d_x, \Delta d_y)$. Hence if the same spatial filter, namely $F(\ )$, is used, the 3D filters in both layers become identical and will be referred to as $H(\Omega)$ hereafter. Let $\tilde{e}_b(t) = e_b(t) - e'_b(t)$, the residue signal between the MCP error signal in the base layer and its quantized version. It can be shown that the mismatch signal $\{\psi\}$ as a function of $\{\tilde{e}_b\}$ and $\{n_e\}$ has exactly the same formulation as that of $\{e_b\}$ as a function of $\{s\}$ and $\{n_b\}$, except that $\alpha H(\Omega)$ serves as the 3D filter [6]. Thus, we obtain an alternative diagram for LPLC as in Fig. 2, which is more amenable to our theoretical analysis.

Fig. 2. Alternative block diagram of LPLC

Fig. 3. Block diagram of LPLC for the EL without drift
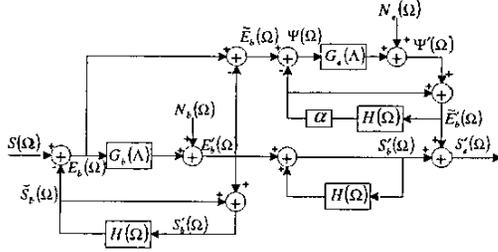
3. RATE DISTORTION ANALYSIS OF LPLC

3.1. Quantization Noise Modeling for 2D Image Coding

Given a 2D image $\{s\}$, we model the quantization noise $\{q\}$ as an additive independent signal whose variance is

$$\sigma_q^2 = \sigma_s^2 2^{-\beta R_s}, \tag{1}$$

where σ_s^2 denotes the variance of the original signal, β denotes the parameter related to the 2D image coding efficiency, and R_s is the data rate in unit of bits/pixel used to encode $\{s\}$ [7].

3.2. Rate Distortion Analysis of LPLC Using Quantization Noise Modeling

Here we use the quantization noise model given by (1) for the 2D image coding. This is different than the optimum forward channel model we used in [6]. We specify three types of data rates: the data rate used by the base layer, R_b, the minimum data rate used by both layers, $R_{e,min}$, and the maximum data rate used by both layers, $R_{e,max}$, which satisfy $R_b < R_{e,min} < R_{e,max}$. We define an MCP rate as the data rate that is incorporated in the MCP step. Thus R_b is the MCP rate in the base layer, and $(R_{e,min} - R_b)$ the MCP rate in the enhancement layer. Two scenarios are considered: with and without drift in the enhancement layer. We assume no drift in the base layer for both scenarios. If the decoded data rate is denoted as $R_{e,dec}$, the above two scenarios correspond to the circumstances where $R_b \leq R_{e,dec}^I < R_{e,min}$ and $R_{e,min} \leq R_{e,dec}^{II} \leq R_{e,max}$ respectively.

3.2.1. The Rate Distortion Function for the Base Layer

The block diagram of LPLC without drift is shown in Fig. 3. We have $e_b' = e_b + q_b$, where $\{q_b\}$ denotes the quantization noise in the base layer with variance as

$$\sigma_{q_b}^2 = \sigma_{e_b}^2 2^{-\beta R_b}, \tag{2}$$

where $\sigma_{e_b}^2$ denotes the variance of $\{e_b\}$. Let $\Phi_{e_b e_b}(\ )$ denote the 2D power spectral density (PSD) of $\{e_b\}$. Similar to [2], we derive

$$\Phi_{e_b e_b}(\ ) = \Phi_{ss}(\ )\left[1 - 2\text{Re}\{F(\ )P(\ )\} + |F(\ )|^2\right] + \Phi_{q_b q_b}(\ )|F(\ )|^2, \tag{3}$$

where $\Phi_{ss}(\ )$ and $\Phi_{q_b q_b}(\ )$ denote the 2D PSD of $\{s\}$ and $\{q_b\}$ respectively, and $\text{Re}\{\ \}$ denotes the real part of a complex function. If all the quantization noise is assumed to be white, we have
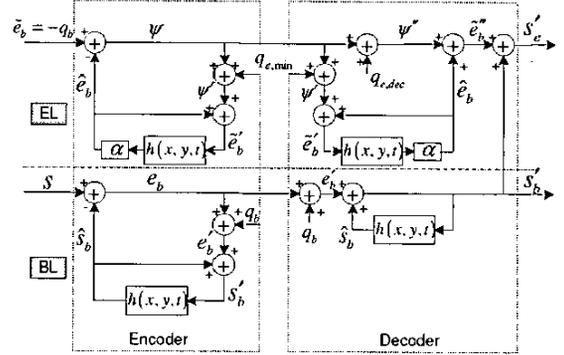
$$\Phi_{q_b q_b}(\ ) = \sigma_{q_b}^2. \text{ Since } \sigma_{e_b}^2 = 1/(4^{-2})\iint_\Lambda \Phi_{e_b e_b}(\ )d\ .$$

$$\sigma_{e_b}^2 = \frac{s}{1 - 2^{-\beta R_b}\ f}, \tag{4}$$

and

$$\sigma_{q_b}^2 = \frac{s}{1 - 2^{-\beta R_b}\ f} 2^{-\beta R_b} = \frac{s}{2^{\beta R_b} - f}, \tag{5}$$

where

$$s \triangleq \frac{1}{4^{-2}}\iint_\Lambda \Phi_{ss}(\ )\left[1 - 2\text{Re}\{F(\ )P(\ )\} + |F(\ )|^2\right]d\ , \tag{6}$$

$$f \triangleq \frac{1}{4^{-2}}\iint_\Lambda |F(\ )|^2 d\ . \tag{7}$$

From Fig. 3, the reconstruction error of the base layer is $r_b = s_b' - s = e_b' - e_b = q_b$. Hence the distortion of the base layer in the mean square error (MSE) sense is

$$D_b(R_b) = Var\{r_b\} = \sigma_{q_b}^2 = \frac{s}{2^{\beta R_b} - f} \triangleq \sigma_b^2. \tag{8}$$

The signal-to-noise-ratio (SNR) of the base layer in dB is

$$SNR_b(R_b) = 10\log_{10}\left(\frac{\sigma_s^2}{\sigma_b^2}\right). \tag{9}$$

3.2.2. The Rate Distortion Function for the Enhancement Layer

Scenario I: The enhancement layer of LPLC is decoded above the MCP rate, namely $R_{e,min} \leq R_{e,dec}^I \leq R_{e,max}$.

As shown in Fig. 3, the enhancement layer in LPLC encodes $\{\tilde{e}_b\}$, where $\tilde{e}_b = s - s_b' = e_b - e_b' = -q_b$. The mismatch signal is $\psi = \tilde{e}_b - \hat{e}_b$, which is quantized to $\{\psi'\}$, where $\psi' = \psi + q_{e,min}$. $\{q_{e,min}\}$ denotes the quantization noise in the enhancement layer within the MCP loop, whose variance is

$$\sigma_{q_{e,min}}^2 = \sigma_\psi^2 2^{-\beta(R_{e,min} - R_b)}, \tag{10}$$

where σ_ψ^2 denotes the variance of $\{\psi\}$. Similar to (3), we derive the 2D PSD of the mismatch signal as

$$\Phi_{\psi\psi}(\ ) = \Phi_{q_b q_b}(\ )\left[1 - 2\alpha\text{Re}\{F(\ )P(\ )\} + \alpha^2|F(\ )|^2\right] + \alpha^2\Phi_{q_{e,min}q_{e,min}}(\ )|F(\ )|^2, \tag{11}$$

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and assume the PSD of $\{q_{e,\min}\}$ as $\Phi_{q_{e,\min}q_{e,\min}}(\ ) = \sigma_{q_{e,\min}}^2$. Thus

$$\sigma_\psi^2 = \frac{\sigma_{q_b}^2(1 - 2\alpha\ f_p + \alpha^2\ f)}{1 - 2^{-\beta(R_{e,\min}-R_b)}\alpha^2\ f}, \tag{12}$$

and

$$\sigma_{q_{e,\min}}^2 = \frac{\sigma_{q_b}^2(1 - 2\alpha\ f_p + \alpha^2\ f)}{2^{\beta(R_{e,\min}-R_b)} - \alpha^2\ f}, \tag{13}$$

where $\sigma_{q_b}^2$ is obtained by (5), f by (7), and

$$f_p \triangleq \frac{1}{4\ ^2}\iint_\Lambda \mathrm{Re}\{F(\ )P(\ )\}d\ . \tag{14}$$

At the encoder in Fig. 3, $\{\tilde{e}_b\}$ is reconstructed as $\{\tilde{e}_b'\}$. At the decoder, since no drift occurs in the enhancement layer, an identical MCP step is included and hence the same MCP signal $\{\hat{e}_b\}$ attained. To reconstruct $\{\tilde{e}_b\}$, however, a different quantization procedure might be used, where the quantization noise is $\{q_{e,\mathrm{dec}}\}$ with variance as

$$\sigma_{q_{e,\mathrm{dec}}}^{2\ (I)} = \sigma_\psi^2 2^{-\beta(R_{e,\mathrm{dec}}^I - R_b)}, \tag{15}$$

where σ_ψ^2 is given by (12). Note that $\sigma_{q_{e,\mathrm{dec}}}^{2\ (I)} \leq \sigma_{q_{e,\min}}^2$ since $R_{e,\mathrm{dec}}^I \geq R_{e,\min}$. This results in a second quantized version of the mismatch signal, $\psi'' = \psi + q_{e,\mathrm{dec}}$, and $\{\tilde{e}_b\}$ is reconstructed as $\tilde{e}_b'' = \psi'' + \hat{e}_b$. The decoded video signal from both layers is then obtained as $s_e' = s_b' + \tilde{e}_b''$.

The reconstruction error of the enhancement layer is $r_e^I = s_e' - s = (s_e' - s_b') + (s_b' - s) = \tilde{e}_b'' + r_b$. Since $\tilde{e}_b'' - \tilde{e}_b = \psi'' - \psi = q_{e,\mathrm{dec}}$, we have $r_e^I = \tilde{e}_b + q_{e,\mathrm{dec}} + r_b = -q_b + q_{e,\mathrm{dec}} + q_b = q_{e,\mathrm{dec}}$. Hence the distortion of the enhancement layer in the MSE sense, as a function of the three types of data rates we specified, is

$$D_e^I(R_b, R_{e,\min}, R_{e,\mathrm{dec}}^I) = Var\left\{r_e^I\right\} = \sigma_{q_{e,\mathrm{dec}}}^{2\ (I)} \triangleq \sigma_e^{2(I)}$$
$$= \frac{1 - 2\alpha\ f_p + \alpha^2\ f}{2^{\beta(R_{e,\min}-R_b)} - \alpha^2\ f} 2^{-\beta(R_{e,\mathrm{dec}}^I - R_{e,\min})} D_b(R_b). \tag{16}$$

The SNR of the enhancement layer in dB then is

$$\mathrm{SNR}_e^I(R_b, R_{e,\min}, R_{e,\mathrm{dec}}^I) = 10\log_{10}\left(\frac{\sigma_s^2}{\sigma_e^{2(I)}}\right). \tag{17}$$

Scenario II: The enhancement layer of LPLC is decoded below the MCP rate, namely $R_b \leq R_{e,\mathrm{dec}}^{II} < R_{e,\min}$.

As shown in Fig. 4, since drift occurs to the enhancement layer, the signal applied to the MCP loop at the decoder is no longer the same as that at the encoder. The reconstruction error of the enhancement layer is $r_e^{II} = s_e' - s = \tilde{e}_b'' + q_b$. We have $\tilde{e}_b'' = \psi'' * h_d^\alpha = -q_b + q_{e,\min} + \Delta q_{e,\mathrm{dec}} * h_d^\alpha$, where $\Delta q_{e,\mathrm{dec}} \triangleq q_{e,\mathrm{dec}} - q_{e,\min}$, $\{*\}$ denotes the convolution operation, and h_d^α is the inverse Fourier transform of $H_d^\alpha(\Omega) \triangleq 1/(1 - \alpha H(\Omega))$. Thus we have $r_e^{II} = q_{e,\min} + \Delta q_{e,\mathrm{dec}} * h_d^\alpha$.

Under the assumptions that uniform embedded quantization operations are used in the enhancement layer, and drift occurs as a result of the truncation to the bitstream of the enhancement layer, the white signals $\{q_{e,\min}\}$ and $\{\Delta q_{e,\mathrm{dec}}\}$ are approximately uncorrelated with each other, and the variance of $\{\Delta q_{e,\mathrm{dec}}\}$ is approximately as

$$\sigma_{\Delta q_{e,\mathrm{dec}}}^2 = \sigma_{q_{e,\mathrm{dec}}}^{2\ (II)} - \sigma_{q_{e,\min}}^2, \tag{18}$$

where

$$\sigma_{q_{e,\mathrm{dec}}}^{2\ (II)} = \sigma_\psi^2 2^{-\beta(R_{e,\mathrm{dec}}^{II} - R_b)}. \tag{19}$$
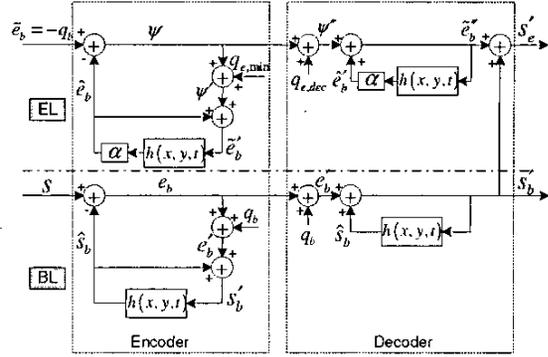
Fig. 4. Block diagram of LPLC for the EL with drift

Using the results in [4] and [6], we have $Var\left\{r_e^{II}\right\}$ as

$$\sigma_{q_{e,\min}}^2 + \sigma_{\Delta q_{e,\mathrm{dec}}}^2\left(\frac{\Delta l}{8\ ^3}\iiint_\Omega E\left[|H_d^\alpha(\Omega)|^2\right]d\Omega\right)$$
$$= \sigma_{q_{e,\min}}^2 + \sigma_{\Delta q_{e,\mathrm{dec}}}^2\left(\frac{1}{4\ ^2}\iint_\Lambda \frac{1}{1 - \alpha^2|F(\ )|^2}d\ \right)$$
$$\triangleq \sigma_{q_{e,\min}}^2 + \sigma_{\Delta q_{e,\mathrm{dec}}}^2\ \frac{\alpha}{d}. \tag{20}$$

Hence, the distortion of the enhancement layer in the MSE sense, as a function of the three types of data rates we specified, is

$$D_e^{II}(R_b, R_{e,\min}, R_{e,\mathrm{dec}}^{II})$$
$$= Var\left\{r_e^{II}\right\} = \sigma_{q_{e,\min}}^2 + \sigma_{\Delta q_{e,\mathrm{dec}}}^2\ \frac{\alpha}{d} \triangleq \sigma_e^{2(II)}$$
$$= \frac{1 - 2\alpha\ f_p + \alpha^2\ f}{2^{\beta(R_{e,\min}-R_b)} - \alpha^2\ f}$$
$$\left(1 + (2^{\beta(R_{e,\min}-R_{e,\mathrm{dec}}^{II})} - 1)\ \frac{\alpha}{d}\right) D_b(R_b). \tag{21}$$

The SNR of the enhancement layer in dB then is

$$\mathrm{SNR}_e^{II}(R_b, R_{e,\min}, R_{e,\mathrm{dec}}^{II}) = 10\log_{10}\left(\frac{\sigma_s^2}{\sigma_e^{2(II)}}\right). \tag{22}$$

4. RATE DISTORTION PERFORMANCE EVALUATION OF LPLC

Similar to [2] and [4], we model the 2D PSD of the input video signal $\Phi_{ss}(\ )$ as

$$\Phi_{ss}(\ ) = \begin{cases} \frac{2\pi}{\omega_0^2}\left(1 + \frac{\omega_x^2 + \omega_y^2}{\omega_0^2}\right)^{-3/2} & |\omega_x| \leq\ f_{sx}, |\omega_y| \leq\ f_{sy} \\ 0 & \text{otherwise} \end{cases} \tag{23}$$

where f_{sx} and f_{sy} denote the sampling frequencies when $\{s\}$ is spatially sampled at the Nyquist rate, and $\omega_0 = \frac{\pi f_{sx}}{42.19} = \frac{\pi f_{sy}}{46.15}$. We model the characteristic function of the estimated motion vector error as

$$P(\ ) = \exp\ -\frac{\sigma_{\Delta d}^2}{2} = \exp\ -\frac{\sigma_{\Delta d}^2}{2}(\omega_x^2 + \omega_y^2)\ , \tag{24}$$

where $\sigma_{\Delta d}^2$ denotes the variance of the estimated motion vector error. We chose $\sigma_{\Delta d}^2 = 0.04/f_{sx}^2$. As shown in Fig. 5, we evaluate the rate distortion performance of LPLC with respect to the
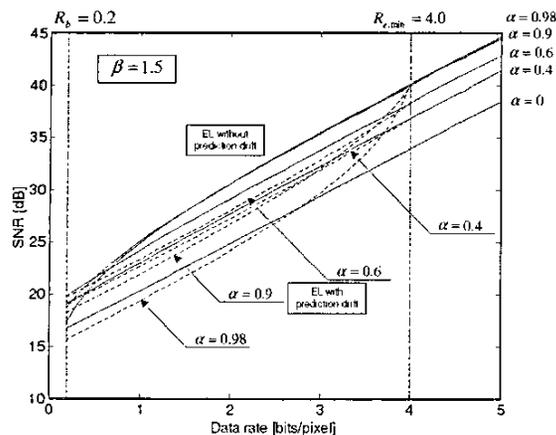
Fig. 5. Evaluation of rate distortion performance of LPLC

leaky factor α according to the closed forms we derived for the two scenarios of LPLC in Section 3.2. Note that in both (16) and (21), we chose $F(\ )$ to be the optimal spatial filter $F_{opt}(\ )$, i.e., $F(\ ) = P^*(\ )$.

Results of *Scenario I for LPLC* are shown in solid lines in Fig. 5, where the enhancement layer (denoted as EL in the figure) does not suffer from drift in LPLC. From (16) and (17), we notice that when R_b and $R_{e,min}$ as well as the leaky factor are fixed, SNR_e^I linearly increases with $R_{e,dec}^I$. We then let $R_{e,dec}^I = R_{e,min}$ in (16), i.e., $\tilde{e}_b' = \tilde{e}_b''$, and vary $R_{e,min}$ between R_b and $R_{e,max}$, as described by the solid lines in Fig. 5. We derive the optimal leaky factor to minimize D_e^I as follows

$$\alpha_{opt} = \frac{\gamma + 1 - \sqrt{(\gamma+1)^2 - 4\gamma\ f}}{2\ f}, \qquad (25)$$

where $\gamma = 2^{\beta(R_{e,min}-R_b)}$. Note that α_{opt} is a function of the MCP rate in the enhancement layer, namely $R_{e,min} - R_b$. When $R_{e,min}$ is sufficiently large, LPLC achieves better performance in the rate distortion sense with increasing leaky factor. A larger leaky factor results in a better decoded quality at the same data rate. For example, when $R_{e,min} = 5$ bits/pixel, SNR_e^I obtains a gain of 3dB by increasing α from 0 to 0.4, or from 0.4 to 0.9. It is interesting to notice that when the enhancement layer MCP rate is small, α_{opt} will be far smaller than 1, implying that a larger leaky factor might yield a less efficient codec, especially when the leaky factor is close to 1. We believe this conforms with the operational results we presented in [9].

Results of *Scenario II for LPLC* are shown by dotted lines in Fig. 5, where the enhancement layer suffers from data rate truncation. We fix $R_{e,min} = 4.0$ while vary $R_{e,dec}^{II}$ between R_b and $R_{e,min}$ according to (21) and (22). It is observed that larger leaky factors yield a larger drop in the rate distortion performance when drift occurs in the enhancement layer, which conforms well with the published operational results. In our closed form expressions, the term $\frac{\alpha}{d}$ in (21) stands for the effect of error propagation when drift occurs. When α approaches 1, we have $\frac{\alpha}{d} \gg 1$. Since $R_{e,dec}^{II} < R_{e,min}$, the term $(2^{\beta(R_{e,min}-R_{e,dec}^{II})} - 1)\ \frac{\alpha}{d}$ in (21) greatly amplifies the distortion with larger leaky factors.

We also evaluated our closed-form expressions with different choices for the parameter β and the three data rates. We varied β

between 0.8 and 1.5 as suggested in [7], and the base layer data rate R_b between 0.05 and 1.0. These rate distortion curves present similar performance as in Fig. 5. We observe that a leaky factor of 0.4-0.6 is a good choice in balancing error resilience performance and coding efficiency for LPLC.

5. CONCLUSIONS

In this paper, we derived the rate distortion functions for LPLC in closed form, using an alternative block diagram of LPLC and a quantization noise model. Two scenarios are considered, where the enhancement layer stays intact at the decoder or suffers from error drift. The theoretical analysis demonstrates that the rate distortion performance of LPLC is closely related to the choice of the leaky factor, which agrees with the operational results published in the literature. A leaky factor between 0.4 and 0.6 is shown to be a good choice.

6. REFERENCES

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