AN IMAGE NORMALIZATION BASED WATERMARKING SCHEME ROBUST TO GENERAL AFFINE TRANSFORMATION

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ABSTRACT

A watermarking algorithm based on image normalization has recently been proposed by Dong and Galatsanos. In this paper, we introduce a different image normalization method for watermarking. Experimental results show that our new image normalization based watermarking algorithm is more robust against the general affine transformation attack. In addition, we show that flipping and aspect ratio information can be obtained using the central moments of both the original image and the watermarked image.

1. INTRODUCTION

The watermarking scheme with moment invariants was described in [1]. This algorithm is shown to be robust against attacks such as flipping, scaling and rotation and is designed to embed only one bit information. The watermark is the mean value of several functions of-the second and third order central moments. In another paper it was shown that flipping, aspect ratio and angle information can be obtained from a function of the central moments [2]. A moment based image watermarking technique was considered in [3]. The algorithm deals with an affine transformation resistent watermark based on image normalization. The algorithm is robust to most affine transformation attacks. In this paper, we propose a different moment based normalization method which may be used for image watermarking. We show that the performance of our algorithm is better than the algorithm in [3] against various geometrical attacks. In addition, we show that flipping and aspect ratio information can be obtained using the central moments of the original image and the watermarked image. This approach is different from the method described in [2].

2. IMAGE NORMALIZATION METHODS

In order to obtain a normalized image, both the central moments and the homogeneous affine transformation matrix have important roles. Let μ_{pq} denote the central moments of the digital image f(x, y) of size $M \times N$ where

$$u_{pq} = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} (x - \bar{x})^p (y - \bar{y})^q f(x, y)$$
(1)

$$\bar{x} = \frac{\sum_{x=0}^{M-1} \sum_{y=0}^{N-1} xf(x,y)}{\sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y)}$$
(2)

$$\bar{y} = \frac{\sum_{x=0}^{M-1} \sum_{y=0}^{N-1} yf(x,y)}{\sum_{x=0}^{M-1} \sum_{x=0}^{N-1} f(x,y)}.$$
(3)

Now we introduce two types of the image normalization methods. The image coordinate may be changed to the another image coordinate through the homogeneous affine transformation matrix A.

The XYS based image normalization

We can decompose the homogeneous affine transformation matrix A into an x-shearing, a y-shearing and an anisotropic scaling matrix, i.e.

$$A \equiv \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} \alpha & 0 \\ 0 & \delta \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \gamma & 1 \end{bmatrix} \begin{bmatrix} 1 & \beta \\ 0 & 1 \end{bmatrix}$$
(4)

with $\alpha, \beta, \gamma, \delta \in \mathbf{C}$. To ensure the uniqueness of the decomposition [4], we require that $det(A) \neq 0$ and $\alpha_{11} \neq 0$. In this case, some work in [4] and [5] use the constraints i.e.

$$\mu_{31} = 0, \ \mu_{13} = 0, \ \mu_{20} = 1, \ \mu_{02} = 1.$$
 (5)

The XSR based image normalization

The homogeneous affine transformation matrix A can be decomposed as an x-shearing, an anisotropic scaling matrix, and a rotation matrix, i.e.

$$A = \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} \alpha & 0 \\ 0 & \gamma \end{bmatrix} \begin{bmatrix} 1 & \beta \\ 0 & 1 \end{bmatrix}$$
(6)

where $\alpha, \gamma, \beta \in \mathbf{R}$ and $\theta \in [0, 2\pi]$. For the uniqueness of the decomposition [4], the condition that $det(A) \neq 0$ is required. In [6], [7] and [8] the affine matrix A is decomposed using the constraints:

$$\mu_{11} = 0, \mu_{20} = \mu_{02} = 1, \ \mu_{30} + \mu_{12} = 0. \tag{7}$$

The matrix A in [8] is different from the matrix A in the XSR based image normalization. In [3] the XSR based image normalization is used for image watermarking.

3. THE GEOMETRICAL INFORMATION USING THE CENTRAL MOMENTS

Lemma 1 [7]: Assume that $f(x_a, y_a)$ is an affine transformed image from f(x, y) by the homogeneous affine transformation matrix

$$\left[\begin{array}{c} x_a \\ y_a \end{array}\right] = \left[\begin{array}{c} a_{11} & a_{12} \\ a_{21} & a_{22} \end{array}\right] \left[\begin{array}{c} x \\ y \end{array}\right],$$

and $\mu_{(\cdot,\cdot)}$ and $\mu'_{(\cdot,\cdot)}$ denote the central moments of the original image and the transformed image, respectively. Then we have the following relationship:

$$\mu'_{pq} = b \sum_{i=0}^{p} \sum_{j=0}^{q} {p \choose i} {q \choose j}$$

$$a_{11}^{i} \cdot a_{12}^{p-i} \cdot a_{21}^{j} \cdot a_{22}^{q-j} \cdot \mu_{i+j,p+q-i-j}$$
(8)

where $b = |a_{11}a_{22} - a_{12}a_{21}|$.

Lemma 2 [4]: Suppose that an image f(x, y) with the image coordinate $(x, y)^t$ is obtained under the normalization constraints $\mu_{31} = 0, \mu_{13} = 0, \mu_{20} = \mu_{02} = 1$. Then another seven images exist under the same normalization constraints. The seven images $f(x_i, y_i)(i = 1, \dots, 7)$ are defined in the image coordinate

$$\begin{pmatrix} \cdot x_i \\ y_i \end{pmatrix} = A_i \begin{pmatrix} x \\ y \end{pmatrix}$$
(9)

where

$$A_{i} = \begin{bmatrix} 0 & \pm 1 \\ \pm 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}.$$
(10)

In the Lemma 2, the seven images $f(x_i, y_i)$ for $i = 1, \dots, 7$ can be obtained by rotating the f(x, y) through $n \times 90^0$ (n = 1, 2, 3)and/or by flipping f(x, y). To avoid the ambiguity of the normalized image, we consider the central moments μ_{12} and μ_{21} . We may be able to determine the rotational information and/or flipping information from the central moments μ_{12} and μ_{21} . Suppose that an image f(x, y) is given and that the image $f'(x_1, y_1)$ is obtained by rotating f(x, y) through 90° counterclockwise. $\mu_{(\cdot, \cdot)}$ and $\mu'_{(\cdot, \cdot)}$ denote the central moments of the given image and the rotated image, respectively. Then the image coordinates have the relationship:

$$\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{pmatrix} x_o \\ y_o \end{pmatrix}.$$
 (11)

Then by Lemma 1, we obtain

$$\dot{\mu_{12}} = \mu_{21}, \quad \dot{\mu_{21}} = -\mu_{12}.$$
 (12)

In addition, suppose that the new image is obtained by rotating the f(x, y) through 180° counterclockwise. Its new central moments are $\mu''_{(\cdot,\cdot)}$. Then the following conditions hold:

$$\mu_{12}^{"} = -\mu_{12}, \quad \mu_{21}^{"} = -\mu_{21}.$$
 (13)

Similarly, suppose that the new image is obtained by rotating f(x, y) through 270° counterclockwise. Its new central moments are $\mu''_{(x,y)}$

and the following conditions hold:

$$\mu_{12}^{\prime\prime\prime} = -\mu_{21}, \ \mu_{21}^{\prime\prime\prime} = \mu_{12}.$$
 (14)

For the new image made by flipping f(x,y) in the horizontal direction, its central moments are $\mu_{(\cdot,\cdot)}^{IV}$ and we obtain

$$\mu_{12}^{IV} = -\mu_{12}, \ \mu_{21}^{IV} = \mu_{21}.$$
 (15)

Finally, for the image made by flipping f(x, y) in the vertical direction, its central moments are $\mu_{(x,y)}^V$ and we obtain

$$\mu_{12}^V = \mu_{12}, \ \mu_{21}^V = -\mu_{21}.$$
 (16)

The aspect ratio information using the central moments

In watermarking, the aspect ratio correction should be made. We may solve this problem using the two central moments μ_{20} and μ_{02} . Suppose that an image f(x, y) with the image coordinate $(x, y)^t$ exists. Then the other image $f(x_s, y_s)$ is defined in the image coordinate

$$\begin{pmatrix} x_s \\ y_s \end{pmatrix} = A_s \begin{pmatrix} x \\ y \end{pmatrix} \equiv \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$
(17)

Indeed, the aspect ratio of the image $f(x_s, y_s)$ is different from that of the image f(x, y) if $a \neq b$. Let μ_{20} and μ_{02} be the central moments from the image f(x, y) and μ_{20}^s and μ_{02}^s be the central moments from the image $f_s(x, y)$. Lemma 1 leads to

$$\mu_{20}^s = a^3 b \mu_{20}, \quad \mu_{02}^s = a b^3 \mu_{02}. \tag{18}$$

Solving for a and b, we obtain

$$a = \sqrt[8]{\frac{\mu_{02}\mu_{20}^{s\,3}}{\mu_{02}^s\mu_{20}^s}}, \quad b = \sqrt[8]{\frac{\mu_{20}\mu_{02}^{s\,3}}{\mu_{20}^s\mu_{02}^s}}.$$
 (19)

If we use the image $f(x_s, y_s)$, the inverse of the matrix A_s in (17) and the equations (19), we may recover the image f(x, y) from $f(x_s, y_s)$. It is noted that if we use the image normalization based watermarking method, we do not need the aspect ratio correction. The reason is that through the image normalization, the normalized image automatically has the form with the corrected aspect ratio.

4. A NEW IMAGE NORMALIZATION BASED WATERMARKING METHOD

We use the XYS image normalization for image watermarking. Instead of using the constraints:

$$\mu_{31} = \mu_{13} = 0, \ \mu_{20} = \mu_{02} = 1,$$
 (20)

we introduce the new constraints

$$\mu_{31} = \mu_{13} = 0, \ \mu_{20} = \mu_{02} = c, \tag{21}$$

where c > 0. By implementing the the iterative method [9, 5] to the simultaneous constraints $\mu_{31}^{xy} = \mu_{13}^{xy} = 0$, we obtain the real roots β^* and γ^* having the smallest norms. $\mu_{(\cdot,\cdot)}^{xy}$ denote the central moments after the x and y shearing operations. By Lemma 1, we obtain two equations:

$$\mu_{20} = \alpha^3 \delta \mu_{20}^{xy}, \ \ \mu_{02} = \alpha \delta^3 \mu_{02}^{xy}. \tag{22}$$

Combining the results with the constraints $\mu_{20} = \mu_{02} = c$ give us

$$\alpha^* = \sqrt[s]{\frac{\mu_{02}^{xy}c^2}{\mu_{20}^{xy}3}}, \ \delta^* = \sqrt[s]{\frac{\mu_{20}^{xy}c^2}{\mu_{02}^{xy}3}}.$$
 (23)

If we want to control the size of the normalized image, we have a relationship between M and c as follows:

$$c \approx \frac{M^4}{(1+|\beta^*|)^4(1+|\gamma^*|)^4} \min(\sqrt{\frac{\mu_{02}^{xy\,3}}{\mu_{20}^{xy}}}, \sqrt{\frac{\mu_{20}^{xy\,3}}{\mu_{02}^{xy}}}). \tag{24}$$

In the above equation, the normalized image has the size of ${\cal M}$ times the size of the original image.

Now, we present the watermarking algorithm based on the image normalization. We use the XYS image normalization for the watermarking. The procedures for embedding the watermark bits are as follows:

Step 1. From the original image, we save the values for μ_{12} and μ_{21} . By applying the image normalization method to the original image, we obtain the normalized image and save the homogeneous affine transformation matrix for the image normalization.

Step 2. Choose the constant c so that the size of the normalized image may be fixed.

Step 3. We embed the set of 50 bits in the normalized image. We obtain the discrete cosine transform version of the normalized image. With private keys, a set of 50 orthogonal sequences are produced and the elements of each sequence are equal to -1 or 1. To the mid range frequency coefficients, we add the weighted orthogonal sequence if the bit to be embedded is 1. Otherwise, we subtract the weighted orthogonal sequence from the DCT coefficients. The weighted orthogonal sequence means the orthogonal sequence multiplied by the weighted watermark strength α .

Step 4: We restore the watermarked image by inverting the normalized image through the inverse of the homogeneous affine transformation matrix in Step 1.

The watermark decoding process is as follows:

Step 1: Comparing the central moments μ_{21} and μ_{12} for the watermarked image and those for the original image, we can find the angle rotational information as described in Section 3.

Step 2: Using the rotational information in Step 1, we can adjust the watermarked image as similar as the original image with respect to the angle orientation $(90^0 \times n, (n = 1, 2, 3))$ and the flipping condition.

Step 3:We find the normalized version from the watermarked image.

Step 4: With the correlation, we determine whether the detected bit is 1 or not in the dct domain. It is noted that we do not need the original image.

5. EXPERIMENTS

In this experiment, the central moments were calculated after normalizing the image to have a maximum of 1 instead of 255. The orthogonal sequences are obtained using a pseudo random uniform distribution. Our XYS normalization based watermarking algorithm is compared with the scheme in [3] on five images using Stirmark 3.1 [10] and attacks described in [1]. The original image, the watermarked image using the XYS image normalization, two normalized images with the XYS image normalization and the XSR image normalization [3] for the Peppers image are

Table 1. Test Results of Stirmark 3.1 [10] and Attacks in [1]

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Fig. 1. (a) The Original Image (b) The Watermarked Image Using the Proposed Algorithm (c) The Normalized Image Using the XYS Method (d)The Normalized Image Using the XSR Method

shown in Fig. 1. As shown in Table 1, the results of the % bit error rate, which means the ratio between the number of the incorrectly decoded bits divided by the total number of embedded bits, are shown. In Table 1, the first row represents the watermarking method: the label "1" means the proposed watermarking method and the label "2" means the algorithm in the paper [3]. For the second row in Table 1, the labels "L","M","V", "P" and "Z" represent the images of Lena, Motorcycles, Valley, Peppers and the Zelda, respectively.

In Table I, the explanation of the first column is as follows:

• From the third row to the 12th row, the results against the attacked angles followed by the cropping are shown. The attacked angles are the values from -1.25 to 1.25 by the increment 0.25. • The labels "11","12" and "13" mean the affine transformation followed by the cropping. The affine transformation matrices are 1.011 0.013 1.007 0.01 1.013 0.008 and 0.009 1.011 0.011 1.008 0.011.012respectively. • The label "s5,5" represents the shearing 5% in the horizontal direction and the shearing 5% in the vertical direction followed by the cropping. • The label "r1.1" represents the first row and the first column removal. • The label "c50" represents the 50 % center cropping. •The labels "sha", "gau" and "med" mean the sharpening filtering, Gaussian filtering and the median 3×3 filtering, respectively. The labels "sal" and "gan" mean the salt and pepper noise with noise intensity 0.02, and the Gaussian noise with zero mean and variance 5×10^{-4} , respectively. •The label "sc0.4" represents the scaling by factor 0.4. • The label "ax.7" represents the scaling by factor 0.7 in the horizontal direction and no change in the vertical direction. "ay.9" represents the scaling by factor 0.9 in the vertical direction and no change in the horizontal direction. • The label "j50" represents the JPEG compression with the quality factor 50. • The label "flplr" represents the flip in the horizontal direction.

We choose the DCT coefficients from 52428th to the 78643th val-

ues in the zig-zag one dimensional alignment. The sizes of the original image and the normalized image are fixed to 512×512 . In Table 1, for each image, we select the same watermark strength α for our algorithm and the scheme in [3] so that the invisibility may be guaranteed. In summary, it is observed that, on an average, our algorithm performs better than the algorithm in [3] against every attack except the flipping in the horizontal direction. In particular, our algorithm performs much better than the work in [3] against such attacks as the crop after the angle rotation, linear geometrical transformation followed by crop, the center cropping, and salt and pepper noise attack. It is noted that the normalized image for our algorithm is larger than that of the work in [3] as shown in Fig. 1.

6. CONCLUSION

In this paper, the XYS image normalization for the image watermarking application has been proposed. Experimental works have shown that the XYS image normalization based watermarking method is more robust than the work in [3] against the general affine transformation attack. In addition, we have shown that the flipping and the aspect ratio information can be obtained using the central moments of the original image and the watermarked image.

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