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***Abstract*— Sensor based surveillance of geographical regions is interesting for applications such as military reconnaissance, environment tracking, and habitat monitoring. For interesting targets to be detected, they must fall within the sensing range of sensors. Static sensors can be used to ensure coverage of whole areas, provided that they have sufficient density and are correctly placed with respect to each other and to the deployment environment. Operating conditions may change, however, which may render the original placement results invalid. To greatly reduce the number of sensors required, and to be robust against dynamic network conditions, mobile sensors can be used to cover given areas of interest (AOI) over time. Moreover, stochastic movement will be effective in overcoming probabilistic and unforeseen changes. In this paper, we develop concepts of network coverage by a set of mobile sensors for given areas of interest, possibly under deadline constraints. Our analytical results characterize the fundamental statistical properties of AOI coverage when sensors move according to an enhanced random waypoint model. Extensive experimental results are reported to verify and illustrate the analytical results.**

I. Introduction

Surveillance of geographical areas by wireless sensors has many interesting applications. For example, the detection of enemy activities in military reconnaissance, the tracking of suspects in a neighborhood, the detection of plumes (e.g., poison gas and radiation) in the environment, and the tracking of animal movement in a habitat. In network surveillance, the *coverage problem* ensures that interesting targets in the observed area are within the sensing range of one or more deployed sensors. Static sensors can be used to provide coverage, provided that they have sufficient density, their sensing range can be accurately characterized (commonly according to a perfect disk assumption), and they are strategically placed with respect to each other and to the deployment environment.

Once the coverage and placement problems have been solved, however, operating conditions may change to render the original results invalid. For example, sensors may fail or their sensing range may weaken, and obstacles may appear that affect a sensor's ability to cover its local area. Mitigating the effects of these unforeseen situations will require yet more sensors (beyond the covering of entire areas) to provide significant redundancy of coverage and hence a safety margin against possible changes.

To greatly reduce the number of sensors required, and to be robust against underspecified or dynamic network conditions, mobile sensors can be used to cover given *areas of interest* (AOI) over time. Moreover, *stochastic* movement will be effective in managing situation uncertainties and non-deterministic changes in the deployment network. This is not unlike how home robots are designed to move to navigate around furniture and vacuum clean whole floor areas, or how robotic mowers try to cover entire lawns despite the unexpected presence of trees, rocks, fences, and other objects.

In this paper, we develop concepts of network coverage by a set of mobile wireless sensors for given AOIs, possibly under given deadline constraints. We present analytical results to characterize various fundamental statistical properties of AOI coverage, when sensors move according to an enhanced random waypoint model – either by design or when carried by mobile hosts engaging in random movement – within a closed network area with boundaries. In particular, we make the following contributions:

- We characterize the spatial distribution of a set of independent sensors in steady state. We show that although sensors are initially placed in any cell with equal probability and are equally likely to pick any cell as the endpoint of each trip, they are likely to concentrate around the middle of the network region in steady state.
- We calculate the expected time until an AOI is first covered or until the AOI is covered entirely, and the expected time a sensor will stay within a cell after entry. We also derive the number of sensors required to cover an AOI with expected time not exceeding a given deadline.
- We present extensive experimental results to verify and illustrate the analytical results.

A. Paper organization

The balance of the paper is organized as follows. In Section II, we present our network and movement models, and derive some basic statistical properties of sensor movement under the models. In Section III, we analyze the coverage problem in a surveillance mobile sensor network. In Section IV, diverse experimental results verifying and illustrating the theoretical analysis are presented. Related work is discussed in Section V. Section VI concludes.

II. System Model

In this section, we define our system model for a mobile sensor network, and state our assumptions. After that, some fundamental characteristics of our model are presented.

A. Network structure

We model our network as a two-dimensional X by Y rectangular area, where X and Y (in distance units) are the width and the height of the network, respectively. (Generalization to 3D space is straightforward.) The whole network is divided into fixed size s by s square regions. Each square region will be called a *cell* hereafter. Cells form a virtual grid over the network area, and each cell has a unique integer cell ID. To simplify boundary conditions, we assume that both X and Y are integral multiples of s. Thus the whole network has $m \times n$ cells, where $m = X/s$ and $n = Y/s$.

B. Movement model

For sensor movement in the network, we consider the widely used random waypoint model [5] enhanced to include a minimum speed specification [10], which ensures meaningful steady state nodal speeds. Under the model, a node moves in a sequence of *trips*. The destination of a trip is chosen uniformly from the network area, and the nodal speed v is chosen uniformly between a minimum speed V_{min} and a maximum speed V_{max}. In a trip, the node moves at constant speed v directly towards destination p. After the mobile node has reached p, it will repeat its decision for the next trip (possibly after a pause time) starting from p. We further assume that (i) the starting point of the first trip is randomly chosen based on uniform distribution and (ii) there is no pause time between the end of a trip and the start of the next trip.[1]

To characterize the stochastic movement, we use the notations defined in Table I.

TABLE I

STOCHASTIC MOVEMENT VARIABLE DEFINITIONS.

Variable	Definition	Type
V_{max}	maximum nodal speed	input parameter
V_{min}	minimum nodal speed	input parameter
V	nodal speed	random variable
T	trip time	random variable
L	trip distance	random variable
$E[V]$	expected speed	statistical property
$E[T]$	expected trip time	statistical property
$E[L]$	expected trip distance	statistical property

C. Statistical properties of nodal movement

This section derives some fundamental statistical properties of our stochastic movement model. They include: (1) the expected distance covered in a trip and the expected time that a trip takes, (2) the distribution of the movement direction taken by the sensor, and (3) the expected *sojourn time* for the sensor to stay in a cell.

[1]The latter assumption is not required in most of our results, and it can be removed in a straightforward manner.

1) **Expected distance and time of a single trip:** Intuitively, the larger the size of the network, the more the expected trip distance $E[L]$. This relationship can be captured formally in the following theorem.

Theorem 1 *Let X and Y denote the width and height of the network. The expected distance for a single trip is*

$$\begin{aligned} E[L] &= \frac{X^2}{6Y}(\ln\frac{1+\sin\alpha}{\cos\alpha} + \frac{\sin\alpha}{\cos^2\alpha}) + \frac{X^3}{15Y^2}(1 - \frac{1}{\cos^3\alpha}) \\ &+ \frac{Y^2}{6X}(\frac{\cos\alpha}{\sin^2\alpha} - \ln\frac{1-\cos\alpha}{\sin\alpha}) + \frac{Y^3}{15X^2}(1 - \frac{1}{\sin^3\alpha}), \end{aligned}$$

where $\alpha = \arctan(Y/X)$.

Proof: Let (x_1, y_1) and (x_2, y_2) denote the starting and ending points of a trip, respectively. Note that both the starting and ending points are uniformly chosen from within the network. Let $Z = |x_1 - x_2|$ and $W = |y_1 - y_2|$, so that the distance of the trip, L, is $\sqrt{Z^2 + W^2}$. To obtain the expected trip distance $E[L]$, we first compute the distributions of Z and W as follows.

The pdf of x_1, or that of x_2, is given by $f(x) = 1/X$. The cdf of Z can then be calculated by:

$$\begin{aligned} F_Z(z) &= P(Z = |x_1 - x_2| < z) \\ &= \int_0^X \int_{x_2-z}^{x_2+z} f(x_1)f(x_2)dx_1dx_2 \\ &= \int_{X-z}^{X+z} f(x_2) \int_{x_2-z}^{X} f(x_1)dx_1dx_2 \\ &\quad + \int_z^{X-z} f(x_2) \int_{x_2-z}^{x_2+z} f(x_1)dx_1dx_2 \\ &\quad + \int_0^z f(x_2) \int_0^{x_2+z} f(x_1)dx_1dx_2 \\ &= \frac{2zX - z^2}{X^2}, \quad 0 \le z \le X. \end{aligned}$$

By differentiating $F_Z(z)$, we obtain the pdf of Z as follows:

$$f_Z(z) = F'_Z(z) = \frac{2}{X} - \frac{2z}{X^2}.$$

Similarly, the pdf of W is given by:

$$f_W(w) = F'_W(w) = \frac{2}{Y} - \frac{2w}{Y^2}.$$

Now, $E[L]$ can be calculated through the joint distribution of Z and W. Let $\alpha = \arctan(Y/X)$. We have

$$\begin{aligned} E[L] &= \int_0^Y \int_0^X \sqrt{z^2+w^2} f_{Z,W}(z,w)dzdw \\ &= \int_0^Y \int_0^X \sqrt{z^2+w^2} f_Z(z)f_W(w)dzdw \\ &= \int_0^Y \int_0^X \sqrt{z^2+w^2} (\frac{2}{X} - \frac{2z}{X^2})(\frac{2}{Y} - \frac{2w}{Y^2})dzdw \\ &= \frac{X^2}{6Y}(\ln\frac{1+\sin\alpha}{\cos\alpha} + \frac{\sin\alpha}{\cos^2\alpha}) + \frac{X^3}{15Y^2}(1 - \frac{1}{\cos^3\alpha}) \\ &+ \frac{Y^2}{6X}(\frac{\cos\alpha}{\sin^2\alpha} - \ln\frac{1-\cos\alpha}{\sin\alpha}) + \frac{Y^3}{15X^2}(1 - \frac{1}{\sin^3\alpha}). \end{aligned}$$

This completes the proof of Theorem 1. □

For the expected time $E[T]$ of a single trip, it should be related to (i) the size of the network and (ii) the input speed constraints. One might think that $E[T] = E[L]/E[V] = E[L]/((V_{max} + V_{min})/2)$. However, this is not correct as there is correlation between the random variables T and V. Theorem 2 gives the correct expression for $E[T]$ based on the input parameters.

Theorem 2 *The expected time of a single trip can be expressed as*

$$\begin{aligned} E[T] &= E[L]E[V^{-1}] \\ &= \frac{\ln(V_{max}/V_{min})}{V_{max}-V_{min}}\left[\frac{X^2}{6Y}(\ln\frac{1+\sin\alpha}{\cos\alpha}+\frac{\sin\alpha}{\cos^2\alpha})\right. \\ &+ \frac{X^3}{15Y^2}(1-\frac{1}{\cos^3\alpha})+\frac{Y^2}{6X}(\frac{\cos\alpha}{\sin^2\alpha}-\ln\frac{1-\cos\alpha}{\sin\alpha}) \\ &+ \left.\frac{Y^3}{15X^2}(1-\frac{1}{\sin^3\alpha})\right], \quad \textit{where } \alpha = \arctan(Y/X). \end{aligned}$$

Proof: Since (i) $E[T] = E[LV^{-1}]$ and (ii) the random variables L and V are independent of each other, we have $E[T] = E[L]E[V^{-1}]$. The value of $E[V^{-1}]$ can be calculated by:

$$\begin{aligned} E[V^{-1}] &= \int_{V_{min}}^{V_{max}} \frac{1}{v} f(v)dv = \int_{V_{min}}^{V_{max}} \frac{1}{v(V_{max}-V_{min})}dv \\ &= \frac{\ln(V_{max}/V_{min})}{V_{max}-V_{min}}. \end{aligned}$$

Theorem 2 follows by combining the above equation with Theorem 1. □

2) **Expected trip direction:** Although the destination of each trip is uniformly distributed within the network area, Theorem 3 shows that the distribution of the movement direction is not uniform, and gives an explicit formula for the distribution.

Theorem 3 *The expected movement direction at a given position (x, y) can be expressed as*

$$\begin{aligned} &E[\delta|(x,y)] = \frac{y^2}{2XY}\left[\ln\frac{\sin(\delta_1+\eta)}{\sin\eta} - \frac{\delta_1}{\tan(\delta_1+\eta)}\right] + \\ &\frac{(X-x)^2}{2XY}\left[\ln\frac{\cos(\delta_2+\eta)}{\cos(\delta_1+\eta)} + \delta_2\tan(\delta_2+\eta) - \delta_1\tan(\delta_1+\eta)\right] \\ &+ \frac{(Y-y)^2}{2XY}\left[\ln\frac{\sin(\delta_3+\eta)}{\sin(\delta_2+\eta)} + \frac{\delta_2}{\tan(\delta_2+\eta)} - \frac{\delta_3}{\tan(\delta_3+\eta)}\right] \\ &+ \frac{x^2}{2XY}\left[\ln\frac{\cos\eta}{\cos(\delta_3+\eta)} + 2\pi\tan\eta - \delta_3\tan(\delta_3+\eta)\right], \end{aligned}$$

where $\eta = \arctan(y/x)$, $\delta_1 = \pi - \eta - \arctan(y/(X-x))$, $\delta_2 = \pi - \arctan((X-x)/y) - \arctan((X-x)/(Y-y)) + \delta_1$, *and* $\delta_3 = \pi - \arctan((Y-y)/(X-x)) - \arctan((Y-y)/x) + \delta_2$.

Proof: The distribution of the movement direction in a $2a$ by $2a$ square area is given in [3]. We extend this result to a network area with dimensions X by Y. Let δ be the the movement angle at the given point (x, y) as defined in Figure 1. The pdf of δ is given by:

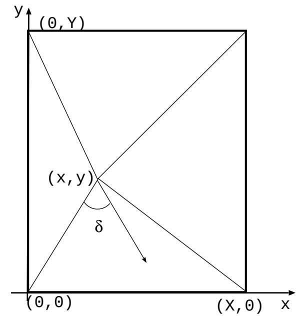


Fig. 1. The definition of δ in a $X \times Y$ area.

$$f(\delta|(x,y)) = \begin{cases} \frac{y^2\csc^2(\delta+\eta)}{2XY}, & 0 \le \delta \le \delta_1; \\ \frac{(X-x)^2\sec^2(\delta+\eta)}{2XY}, & \delta_1 \le \delta \le \delta_2; \\ \frac{(Y-y)^2\csc^2(\delta+\eta)}{2XY}, & \delta_2 \le \delta \le \delta_3; \\ \frac{x^2\sec^2(\delta+\eta)}{2XY}, & \delta_3 \le \delta \le 2\pi. \end{cases}$$

Then the corresponding cdf of δ is:

$$F(\delta|(x,y)) = \begin{cases} \frac{y^2(\cot\eta-\cot(\delta+\eta))}{2XY}, & 0 \le \delta \le \delta_1; \\ \frac{(X-x)^2(\tan(\delta+\eta)-\tan(\delta_1+\eta))}{2XY}, & \delta_1 \le \delta \le \delta_2; \\ \frac{(Y-y)^2(\cot(\delta_2+\eta)-\cot(\delta+\eta))}{2XY}, & \delta_2 \le \delta \le \delta_3; \\ \frac{x^2(\tan(\delta+\eta)-\tan(\delta_3+\eta))}{2XY}, & \delta_3 \le \delta \le 2\pi. \end{cases}$$

Given the pdf of δ at location (x, y), the desired expected movement direction is equal to

$$\begin{aligned} &E[\delta|(x,y)] = \int_0^{2\pi} \delta f(\delta|(x,y))d\delta \\ &= \int_0^{\delta_1} \frac{y^2\csc^2(\delta+\eta)}{2XY}\delta d\delta + \int_{\delta_1}^{\delta_2} \frac{(X-x)^2\sec^2(\delta+\eta)}{2XY}\delta d\delta \\ &+ \int_{\delta_2}^{\delta_3} \frac{(Y-y)^2\csc^2(\delta+\eta)}{2XY}\delta d\delta + \int_{\delta_3}^{2\pi} \frac{x^2\sec^2(\delta+\eta)}{2XY}\delta d\delta \\ &= \frac{y^2}{2XY}\left[\ln\frac{\sin(\delta_1+\eta)}{\sin\eta} - \frac{\delta_1}{\tan(\delta_1+\eta)}\right] + \\ &\frac{(X-x)^2}{2XY}\left[\ln\frac{\cos(\delta_2+\eta)}{\cos(\delta_1+\eta)} + \delta_2\tan(\delta_2+\eta) - \delta_1\tan(\delta_1+\eta)\right] \\ &+ \frac{(Y-y)^2}{2XY}\left[\ln\frac{\sin(\delta_3+\eta)}{\sin(\delta_2+\eta)} + \frac{\delta_2}{\tan(\delta_2+\eta)} - \frac{\delta_3}{\tan(\delta_3+\eta)}\right] \\ &+ \frac{x^2}{2XY}\left[\ln\frac{\cos\eta}{\cos(\delta_3+\eta)} + 2\pi\tan\eta - \delta_3\tan(\delta_3+\eta)\right]. \end{aligned}$$

This completes the proof of Theorem 3. □

In the next section, we will further show how the distribution of the movement direction affects the coverage by a mobile sensor.

3) **Expected cell sojourn time:** Once a mobile sensor enters a cell, it may stay in the covered cell for an amount of time called the *sojourn time* before leaving. The sojourn time depends on both the cell size and the movement model. We now derive the expected cell sojourn time.

To begin with, we approximate the network cell by a circle that has the same area (see Figure 2). The radius of the circle, denoted by R, is therefore

$$R = \frac{s}{\sqrt{\pi}} \approx 0.56s.$$

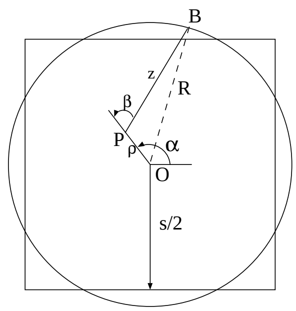


Fig. 2. Approximation of cell as a circle.

To calculate the expected sojourn time for a cell, we consider two movement scenarios of a sensor in a single trip: (1) The sensor starts or finishes the trip inside the cell; and (2) The sensor passes through the cell during the trip. Theorem 4 gives the expected sojourn time in each of the two cases.

Theorem 4 *Assume that each network cell is a circle with radius R. Within a single trip, the expected sojourn time*

(i) *when the sensor is inside the starting cell (or inside the ending cell) is*

$$\frac{8R}{3\pi(V_{max} - V_{min})} \ln \frac{V_{max}}{V_{min}};$$

(ii) *when the sensor is not inside the starting cell or the ending cell (i.e., when the sensor passes through a cell in the trip) is*

$$\frac{4R}{\pi(V_{max} - V_{min})} \ln \frac{V_{max}}{V_{min}}.$$

Proof: We first calculate the expected sojourn time of Case (i). Let T_i denote the sojourn time that a mobile sensor is inside the starting cell, and let V denote the nodal speed. The joint pdf of T_i and V is given by

$$f_{T_i,V}(t,v) \quad = \quad |v| f_{Z,V}(z,v).$$

where Z is the distance from the sensor to the cell boundary.

The pdf of T_i can be calculated using the above joint pdf as follows: Let $C_1 = \dfrac{2}{\pi R^2(V_{max}-V_{min})}$ and $C_2(t) = \dfrac{8R}{3\pi t^2(V_{max}-V_{min})}$. Then, we have

$$f_{T_i}(t) = \int_{-\infty}^{\infty} f_{T_i,V}(t,v)dv = \int_{-\infty}^{\infty} |v| f_{Z,V}(z,v)dv$$

$$= \begin{cases} C_1 \int_{V_{min}}^{V_{max}} v\sqrt{R^2 - (\frac{tv}{2})^2}dv, & 0 \le t \le \frac{2R}{V_{max}}; \\ C_1 \int_{V_{min}}^{2R/t} v\sqrt{R^2 - (\frac{tv}{2})^2}dv, & \frac{2R}{V_{max}} \le t \le \frac{2R}{V_{min}}; \\ 0, & t \ge \frac{2R}{V_{min}}. \end{cases}$$

$$= \begin{cases} C_2(t) \left([1 - (\frac{tV_{min}}{2R})^2]^{3/2} - [1 - (\frac{tV_{max}}{2R})^2]^{3/2}\right), & 0 \le t \le \frac{2R}{V_{max}}; \\ C_2(t) \; [1 - (\frac{tV_{min}}{2R})^2]^{3/2}, & \frac{2R}{V_{max}} \le t \le \frac{2R}{V_{min}}; \\ 0, & t \ge \frac{2R}{V_{min}}. \end{cases}$$

Then, the expected sojourn time in Case (i) becomes

$$\begin{aligned} E[T_i] &= \int_{-\infty}^{\infty} t f_T(t)dt \\ &= \frac{8R}{3\pi(V_{max} - V_{min})} \times \\ &\quad \left\{ \int_0^{\frac{2R}{V_{max}}} \frac{1}{t} \left[(1 - (\frac{tV_{min}}{2R})^2)^{3/2} - (1 - (\frac{tV_{max}}{2R})^2)^{3/2} \right] dt \right. \\ &\quad \left. + \int_{\frac{2R}{V_{max}}}^{\frac{2R}{V_{min}}} \frac{1}{t} (1 - (\frac{tV_{min}}{2R})^2)^{3/2} dt \right\} \\ &= \frac{8R}{3\pi(V_{max} - V_{min})} \ln \frac{V_{max}}{V_{min}}. \end{aligned}$$

To calculate the expected sojourn time in Case (ii), let T_c denote the sojourn time when the sensor is in a cell other than the starting and the ending cells (i.e., when the sensor is crossing a cell in the trip). Let Z_c be the length of chord where the path of the sensor intersects the cell. The pdf of Z_c is

$$f_{Z_c}(z) = \frac{2}{\pi\sqrt{(2R)^2 + z^2}}, \quad 0 \le z \le 2R.$$

Then, the expected chord length is

$$E[Z_c] = \int_0^{2R} z f_Z(z)dz = \frac{4R}{\pi}.$$

Also, in the proof of Theorem 2, we know that

$$E[V^{-1}] = \frac{\ln(V_{max}/V_{min})}{V_{max} - V_{min}}.$$

Then, the expected sojourn time in Case (ii) becomes

$$E[T_c] = E[Z_c]E[V^{-1}] = \frac{4R}{\pi(V_{max} - V_{min})} \ln \frac{V_{max}}{V_{min}}.$$

This completes the proof of Theorem 4. □

To compute the overall expected cell sojourn time, we need to consider both the "inside a cell" and "crossing a cell" cases. Denoting the probabilities for each case to happen by P_{inside} and $P_{crossing}$, respectively, we have the following corollary.

Corollary 1 *The expected cell sojourn time $E[T_s]$ is*

$$E[T_s] = P_{inside}E[T_i] + P_{crossing}E[T_c].$$

As an example, for a network divided into 5 by 5 cells, P_i and P_c are 0.454 and 0.546, respectively. Further discussions on the cell sojourn time will appear in Section IV.

III. Coverage in Surveillance Mobile Sensor Network

When a surveillance sensor moves in a network, it monitors an area within its sensing range. In this section, we give some interesting results on the sensor coverage. They include: (1) How long does it take before a sensor will cover a target cell? (2) How long does it take before a sensor will cover any cell, or all the cells, in an *area of interest* (AOI)? (3) Given the goal to cover a target cell or an AOI within an expected deadline constraint, how many sensors are required?

In the following, we say that a sensor *covers* some cell i whenever the sensor enters cell i from another cell. This happens when a trip passes through cell i, or cell i is the trip's destination. Hence, if a sensor enters a cell, leaves it, and enters it again, we say that the cell is covered twice by the sensor.

A. Coverage of a cell

We are interested in how long it takes before a given cell is covered. We first consider the case in which we naively (and incorrectly) assume that any cell in the network is equally likely to be the next cell covered, whenever the sensor leaves a current cell. After that, we correct the naive case by refining the probability distribution of the cells that will be covered next, thus obtaining the desired time of coverage.

Coverage in the naive case: In this case, when a mobile sensor leaves a cell, the probability that a given *target cell* is the next cell covered is $1/(mn)$, where m and n are the number of rows and columns in the grid of cells, respectively. Let k denote the number of cells visited by the sensor before the sensor covers the target cell. Theorem 5 gives the expected value of $E[k]$.

Theorem 5 *Assume that each cell is equally likely to be the next one covered when a sensor leaves a current cell. The expected number of cells visited by the sensor before covering a target cell is equal to the total number of cells in the network; i.e., $E[k] = mn$.*

Proof: Let $P\{k\}$ be the probability that the sensor *first* covers the target cell after visiting k other cells. Then we have

$$P\{k\} = (1 - \frac{1}{mn})^{k-1}\frac{1}{mn}, \quad k \geq 1.$$

Then, the expected number of cells visited by the sensor before covering the target cell can be calculated by

$$E[k] = \sum_{k=1}^{\infty} kP\{k\} = \sum_{k=1}^{\infty} k(1 - \frac{1}{mn})^{k-1}\frac{1}{mn} = mn.$$

This completes the proof of Theorem 5. □

Coverage in the refined case: In Theorem 5, we assume that each cell is equally likely to be the next cell covered. This assumption seems reasonable at first glance, since the whole network is partitioned into cells of equal size, and the probability for any cell to be chosen as a trip's destination, according to the random waypoint model, is the same for all the cells.

However, apart from the two end points of a trip, a mobile sensor covers also the intermediate cells along the path. Intuitively, cells near the center of the network are covered more frequently, although the destination of a trip is equally likely to be any cell. The intuition is supported by Theorem 3, which states that the sensor is most likely to move to the center of the network.

Based on the above observation, we make precise our calculation of a sensor's coverage. We first calculate the probability P_i that cell i is the next cell covered when the sensor leaves a cell. To simplify the analysis, we represent each cell by its center. Then, for each cell j, we enumerate all the possible routes to other cells in the network, where a route between two cells is defined as the line segment between the centers of the cells. Each time a cell is intersected by a route, the cell is given a credit of one. Let c_i denote the total credits received by cell i, and C_{total} be the total number of credits given to all the cells. We have $P_i = c_i/C_{total}$. Based on P_i, we can apply Theorem 6 to find the expected number of cells visited before a given cell is covered.

Theorem 6 *Let P_i denote the probability that cell i is the next cell covered when the sensor leaves a current cell. The expected number of cells visited by the sensor before cell i is covered, denoted as $E[k_i]$, is $1/P_i$.*

Proof: Let $P_i\{k\}$ denote the probability that the mobile sensor first covers cell i after visiting k other cells. Then, we have

$$P_i\{k\} = (1 - P_i)^{k-1}P_i, \quad k \geq 1.$$

The expected value of k_i for cell i is then given by

$$E[k_i] = \sum_{k=1}^{\infty} kP_i\{k\} = \sum_{k=1}^{\infty} k(1 - P_i)^{k-1}P_i = \frac{1}{P_i}.$$

This completes the proof of Theorem 6. □

Notice that the expected number of cells visited by the sensor before covering a given target cell i becomes $1/P_i$ instead of $1/(mn)$.

Let $E'[k]$ denote the expected number of cells visited before the target cell is covered. Suppose that all the cells have equal probability to be the target cell. Then, $E'[k]$ is equal to $(1/(mn)) \sum_{i=1}^{mn} (1/P_i)$. In general, if we let $P_{target=i}$ be the probability that cell i is the target cell, then $E'[k]$ becomes:

$$E'[k] = \sum_{i=1}^{mn} P_{target=i}E[k_i] = \sum_{i=1}^{mn} \frac{P_{target=i}}{P_i}$$

We now discuss another interesting question: How long does it take before a sensor will cover the target cell? First, let

T_{target} be the time for a sensor to first cover the target cell. Since it may take several trips before the sensor enters the target cell, we let $P(N_{trip})$ be the probability that the sensor takes exactly N_{trip} trips before entering the target cell, and let $E[T_{N_{trip}}]$ be the expected time of these N_{trip} trips. Then, we have:

$$E[T_{target}] = \sum_{N=1}^{\infty} P(N_{trip}) E[T_{N_{trip}}]$$

In practice, it is difficult to obtain precise values for both $P(N_{trip})$ and $E[T_{N_{trip}}]$. We now give an approximation of $E[T_{target}]$ based on the expected cell sojourn time $E[T_s]$. As the cell sojourn time is the time that a sensor will stay in a cell, the expected travel time before the sensor covers the target is about:

$$E[T_{target}] \approx E[T_s] E'[k]. \tag{1}$$

Also, we may approximate the expected number of trips before the sensor covers the target cell, based on the expected trip time $E[T]$, as follows:

$$E[N_{trip}] \approx \frac{E[T_s]E'[k]}{E[T]}. \tag{2}$$

B. Coverage of an area of interest (AOI)

We extend the coverage results of single cells to the coverage of an *area of interest* (AOI) by a mobile sensor. In general, an AOI consists of one or more cells in the network area that form a target of surveillance. We are interested in two questions: (1) How long will it take before a mobile sensor enters an AOI? (2) How long will it take before the sensor visits all the cells in the AOI.

We answer the first question as follows. As defined in Section III-A, P_i is the probability that cell i is the next cell covered. Then, the probability that the sensor will next cover a cell in the AOI is equal to $P_{AOI} = \sum_{i \in AOI} P_i$. Once P_{AOI} is obtained, we can make use of the following theorem to answer the first question.

Theorem 7 *Given P_{AOI}, the expected number of cells visited by a mobile sensor before it enters any cell in an AOI, denoted as $E[k_{AOI_{1st}}]$, is $1/P_{AOI}$.*

Proof: Let $P_{AOI}\{k\}$ be the probability that the mobile sensor may move to any cell in an AOI after visiting k cells. The expected value of k is:

$$\begin{aligned} E[k_{AOI_{1st}}] &= \sum_{k=1}^{\infty} k(1-P_{AOI})^{k-1} P_{AOI} \\ &= \frac{P_{AOI}}{1-P_{AOI}} \left[\sum_{k=0}^{\infty} k(1-P_{AOI})^k - k(1-P_{AOI})^k |_{k=0} \right] \\ &= \frac{1}{P_{AOI}} \end{aligned}$$

□

By substituting $E[k_{AOI_{1st}}]$ for $E'[k]$ in Equation 1 in Section III-A, we have the expected travel time before covering a cell in the AOI as:

$$E[T_{AOI_{1st}}] \approx E[T_s] E[k_{AOI_{1st}}],$$

and similarly, we may approximate the expected number of trips before the mobile enters the AOI by:

$$E[N_{trip}] \approx \frac{E[T_s]E[k_{AOI_{1st}}]}{E[T]}.$$

We proceed to answer the second question, namely how long does it take for the mobile sensor to cover all the cells in an AOI?

Theorem 8 *Let P_i be the probability for the target cell i to be covered by the sensor at any instance of time. The expected number of cells visited by the sensor*

(i) *before covering all the cells in an AOI is*

$$E[k_{AOI}] \leq \sum_{k=1}^{\infty} k \sum_{i \in AOI} (1-P_i)^{k-1} P_i;$$

(ii) *before covering all the cells in the network is*

$$E[k_{all}] \leq \sum_{k=1}^{\infty} k \sum_{i=1}^{mn} (1-P_i)^{k-1} P_i.$$

Proof: To prove (i), we observe that the term $\sum_{k=1}^{\infty} k \sum_{i \in AOI} (1-P_i)^{k-1} P_i$ is the sum of $E[k_i]$, for all $i \in AOI$, where k_i is the number of cells visited by the sensor before covering cell i. On the other hand, $\sum_{i \in AOI} k_i$ represents the number of cells in the sequence of cells visited before every cell in the AOI is covered. Note that by the time we have visited the last cell in the above sequence, each cell in the AOI has been covered at least once. Thus, $k_{AOI} \leq \sum_{i \in AOI} k_i$, which implies that $E[k_{AOI}] \leq E[\sum_{i \in AOI} k_i] = \sum_{i \in AOI} E[k_i]$, where the latter equality follows from the linearity of expectation. Combining these results, we complete the proof for (i). The result of (ii) follows as a special case of (i), in which the AOI is the whole network area. □

Let T_{AOI} and T_{all} denote the times to cover all the cells in the AOI and the time to cover all the cells in the network, respectively. Based on the expected cell sojourn time $E[T_s]$, we can approximate $E[T_{AOI}]$ by

$$E[T_{AOI}] \approx E[T_s] E[k_{AOI}],$$

and we can approximate $E[T_{all}]$ by

$$E[T_{all}] \approx E[T_s] E[k_{all}].$$

C. Number of sensors for coverage within a deadline

In some applications, we need to cover a target cell or AOI within a deadline constraint D. If D is small (i.e., $E[T_{target}] \geq D$), one sensor is not sufficient, and multiple sensors, say N of them, must be used to reduce the expected coverage time. We assume that the movements of the N sensors are independent and follow the same distribution. Then, we have the following theorem.

Theorem 9 *Suppose that we are given the deadline D, the expected cell sojourn time $E[T_s]$, and the probability P_i for the target cell i to be covered at any instance of time (if there is only one sensor). Then, to cover the target cell i within expected time D, the minimum number of sensors required is about $\lceil E[T_s]/(DP_i) \rceil$.*

Proof: If $N \geq 1/P_i$, the target cell is expected to be covered by at least one sensor at any time. Therefore, we consider the case of $N < 1/P_i$. To obtain the desired approximation in this case, we assume that the sensors move in a coordinated manner, such that they spend $E[T_s]$ time in a cell, and then they visit a new cell at the same time. Under this assumption, the probability for any of the N sensors to cover the target cell after visiting k cells is

$$P_{N_i}\{k\} = (1 - NP_i)^{k-1} NP_i, \quad k \geq 1$$

and the expected number of cells visited by a sensor before one of the N sensors covers the target cell, denoted by k_{N_i}, is

$$E[k_{N_i}] = \sum_{k=1}^{\infty} k(1 - NP_i)^{k-1} NP_i = \frac{1}{NP_i}$$

Hence, the expected time to cover the target cell by any of the N sensors is

$$E[T_{target_N}] \approx E[k_{N_i}]E[T_s] = \frac{E[T_s]}{NP_i}$$

where $E[T_s]$ is the expected cell sojourn time for a sensor.

By setting $E[T_{target}] \leq D$, we have:

$$N_{\min} \approx \left\lceil \frac{E[T_s]}{DP_i} \right\rceil$$

which gives the minimum number of sensors for covering the target cell within expected time D. $\square$

We can similarly compute the expected minimum number of sensors for covering an AOI by a deadline D. We omit the details due to space constraints.

IV. Simulation Results

We first present simulation results on the basic statistical properties of our stochastic movement model. We then present results on the sensor coverage problem. Results are reported as averages over a large number of experimental runs. Error bars are omitted because the standard deviations of the results are small.

A. Expected trip distance

The expected trip distance $E[L]$ is calculated by Theorem 1. To verify our calculations, we perform a simulation experiment with 10,000,000 trips and measure the average trip distance of these trips. Our simulation is run on networks with various cell sizes. In Table II, we show the calculated results and the measured results. Notice that our calculations of $E[L]$ are close to the measured average trip distances.

TABLE II

COMPARISON BETWEEN CALCULATED AND MEASURED $E[L]$ (IN M).

rectangle	calculation	measurement
100x100	52.14	52.14
150x150	78.21	78.21
500x500	260.70	260.63
1000x1000	521.41	520.40
1500x1500	782.11	782.24

B. Distribution of movement direction

In Theorem 3, we show that the distribution of the movement angle δ varies by location in the network. In Figure 3(a), we illustrate the cdf of δ in a given area of 150 m $\times$ 150 m, for different network locations [e.g., $(30, 30)$ refers to the location where the x and y-coordinates are both 30 m]. From the figure, notice that δ shows an approximate uniform distribution at the center of the network but is less uniform when approaching the network boundary.

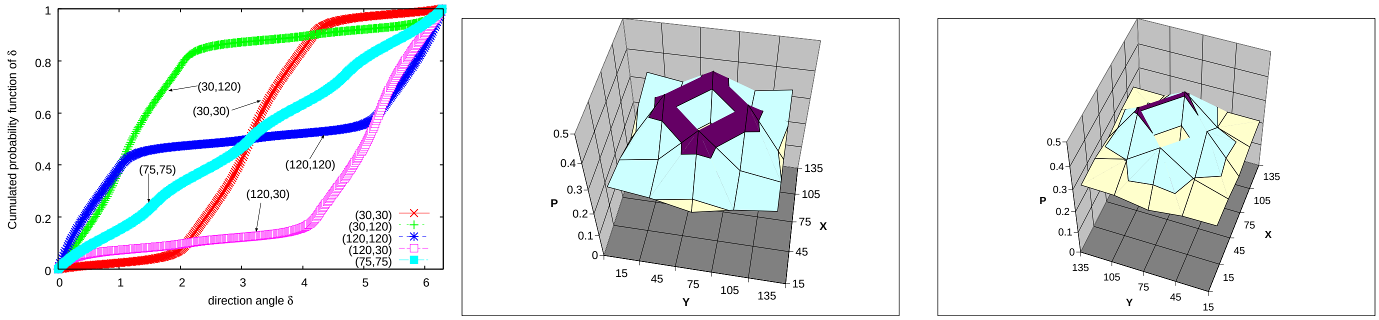
Applying the distribution of the movement direction, we can deduce the probability for a mobile sensor to move towards the target cell, from different current positions. In Figure 3(b) and Figure 3(c), we illustrate an example scenario in which the target cell to be covered is located at the center cell of a 150 m by 150 m network area, where the area is divided into 5×5 cells. The figures show the probabilities for the mobile sensor to move towards the target (center cell) from different current cell positions. We find that the closer the sensor is from the center, the higher the probability that it will move towards the center cell.

C. Expected cell sojourn time

In calculating the expected cell sojourn time in Theorem 4, we have derived the pdf of T_i (where T_i is the sojourn time when the sensor is at the end points of a trip) as a function of three system parameters: the cell size, the maximum nodal speed, and the minimum nodal speed. Here, we illustrate the effect of these parameters on the pdf.

The effect of cell size is shown in Figure 4(a). The three curves in the figure are the corresponding cdfs of T_i when the cell size is 30 m, 50 m, and 100 m, respectively. From the figure, we find that at any fixed probability, when the cell size of the network is smaller, the sojourn time is smaller. This implies that $E[T_i]$ is smaller when the cell size is smaller, which agrees with our calculation.

The effect of maximum nodal speed is shown in Figure 4(b). The three curves in the figure are the corresponding cdfs of T_i when the maximum speed is 5 m/s, 10 m/s, and 15 m/s,

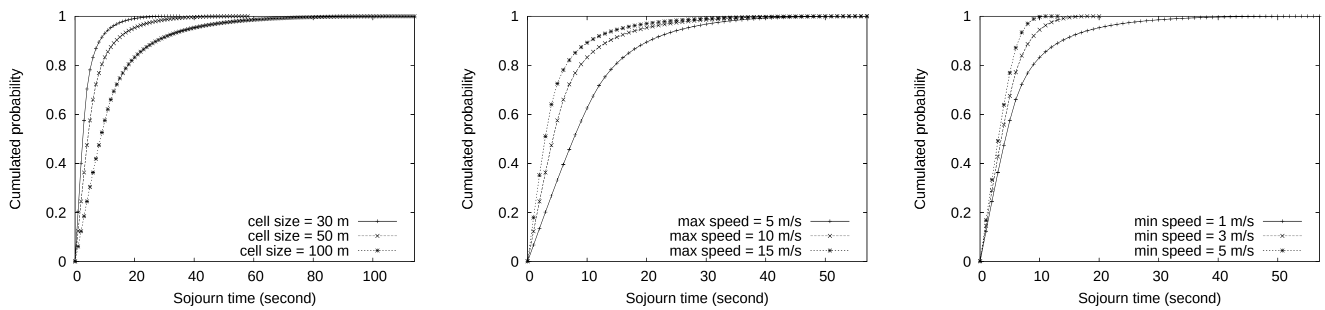


(a) Cumulated probability function of δ.

(b) Calculated probability of sensor moving towards the center cell of the network from different current locations.

(c) Measurement of probability of sensor moving towards the center cell of the network from different current locations.

Fig. 3. Distribution of the movement direction in a 150 m by 150 m network area.



(a) With different cell sizes ($V_{max} = 10$ m/s, $V_{min} = 1$ m/s)

(b) With different maximum speeds (cell size $s = 50$ m, $V_{min} = 1$ m/s)

(c) With different minimum speeds (cell size $s = 50$ m, $V_{max} = 10$ m/s)

Fig. 4. Comparison of the cumulated probabilities of the sojourn time as a function of different system parameters.

respectively. The result implies that $E[T_i]$ is smaller when the maximum speed is higher, which agrees with our calculation.

The effect of minimum nodal speed is shown in Figure 4(c). The three curves in the figure are the corresponding cdfs of T_i when the minimum speed is 1 m/s, 3 m/s, and 5 m/s, respectively. The result implies that $E[T_i]$ is smaller when the minimum speed is higher, which agrees with our calculation.

In Theorem 4, we illustrate how to calculate the expected cell sojourn time. Next, we compare the expected sojourn times $E[T_i]$ and $E[T_c]$ calculated in Theorem 4 with the measured times obtained from simulations. For each setting of (s, V_{max}, V_{min}) shown in Table III (recall that s denotes the cell dimension), we perform an experiment consisting of 100 independent 200,000-second runs to obtain the average sojourn time $T_{measure}$.

In the table, we present the calculated times $E[T_i]$, $E[T_c]$ and $E[T_s]$, the measured time $T_{measure}$, and the maximum possible sojourn time $T_{max} = 2 * R/V_{min}$ for each setting. Note that $E[T_s]$ and $T_{measure}$ are always bounded by $E[T_i]$ and $E[T_c]$, which agrees with the fact that $E[T_s]$ is bounded by $E[T_i]$ and $E[T_c]$ (see Corollary 1).

TABLE III

CALCULATED AND MEASURED EXPECTED SOJOURN TIMES (IN S).

s (m)	V_{max} (m/s)	V_{min} (m/s)	$E[T_i]$	$E[T_c]$	T_{max}	$E[T_s]$	$T_{measure}$
30	10	4	2.70	4.07	8.46	3.44	3.05
50	10	4	4.17	6.25	14.10	5.31	5.07
100	10	4	7.82	11.73	28.21	10.11	10.11
50	5	4	5.85	8.77	14.10	7.65	7.65
50	10	4	4.17	6.25	14.10	5.31	5.07
50	15	4	3.39	5.09	14.10	4.32	3.96
50	10	3	4.63	6.95	18.81	5.89	5.81
50	10	4	4.17	6.25	14.10	5.31	5.07
50	10	5	3.83	5.75	11.28	4.87	4.56

D. Covering a target cell

In this section, we show the results for a mobile sensor to cover a target cell. In the experiments, a 150 m by 150 m network is divided into 25 cells of size 30 m $\times$ 30 m. We report average measurement results over five independent 2,000,000-second simulation runs. First, in Figure 5(a), the probability of the mobile sensor covering each cell is shown by a vertical bar. The corresponding probability obtained through simulation is shown in Figure 6(a). Observe that the calculated results closely match the measurement results. Also, in both figures, notice that the sensor is more likely to visit cells in the center of the network, which implies that the sensor passes through the center area more frequently than the boundary area, as discussed in Section III-A.

We now illustrate the expected number of trips taken by a mobile sensor before it covers a target cell. Figure 5(b) and Figure 6(b) show the calculated and measured results, respectively. Observe the close similarity between the two figures. Also, both sets of results show that the sensor spends fewer trips before covering a cell in the network center than covering a cell in the network boundary.

The calculated and measured results for the expected / average time to cover a target cell are given in Figure 5(c) and Figure 6(c), respectively. These figures are similar to the corresponding figures for the expected number of trips.

Lastly, we present results for coverage of the whole network area. In Theorem 8, we derive the formula for the expected number of cells visited before the whole network area is covered. To verify the formula, we measure the average numbers of cells visited over five 2,000,000-second simulation runs, for different cell sizes. The results are reported in Table IV, which suggest that the derived formula closely matches the actual average values.

TABLE IV
CALCULATED AND MEASURED EXPECTED NUMBER OF CELLS VISITED BEFORE THE WHOLE NETWORK AREA IS COVERED.

grid	calculation	measurement
3x3	45	63
5x5	325	349
7x7	1225	1073

V. Related Work

The sensor coverage problem has attracted much attention recently. The minimum number of nodes to cover a network area is studied in [3], [7], [8]. An important application of the coverage problem is the tracking of moving objects, where wireless sensors form a network and cooperatively collect and exchange tracking information among themselves (e.g., through a cluster-based scheme [2], [9] or a tree-based scheme [11]). Also, a technique is proposed in [12] to dynamically optimize the usage of system resources in collecting information by using *leader* nodes in a sensor network.

The above body of work focuses on the use of static sensors in a network area. Our work considers the coverage problem of *mobile* sensors. Some interesting results on network coverage by mobile sensors are presented in [6]. Compared with their work, ours uses different network and movement models. They consider the network to be an open and infinite space, whereas we consider a closed network area with explicit boundaries. They consider nodal movements in straight lines (similar to a single trip in our case) and optimize the movement angles, whereas we consider movement as a sequence of trips defined by the random waypoint model. These differences require new analysis techniques and can lead to significantly different conclusions (e.g., the non-uniform steady-state spatial distribution of sensors in our case).

The distribution of movement direction in Section II-C.2 has been studied in [3]. Their focus is to solve the handoff problem, and they consider the network area to be a square. Our analysis applies to arbitrary rectangular areas, and corrects a minor (but quantitatively significant) mistake in the corresponding formula in [3]. By analyzing the expected movement direction of mobile sensors, we find that a mobile sensor is most likely to move towards the central area of the network. It is possible to draw a similar conclusion that the mobile nodes will concentrate at the network center by considering the intersection of each infinitesimally small cell with a single trip of the random waypoint model [1]. Our work uses the non-uniform distribution property to derive other statistical properties of the mobility (e.g., the expected trip time) and several important coverage results of general AOIs, which are not the focus in [1].

VI. Conclusion

We have presented fundamental analytical results about sensor movement according to an enhanced form of the widely used random waypoint model. We have demonstrated the relevance of our results by relating them to the problem of area coverage in surveillance mobile sensor networks. We have answered several important performance questions regarding the coverage of an area of interest (AOI) by a set of mobile sensors. Extensive experimental results reported verify and illustrate the accuracy of the analytical results.

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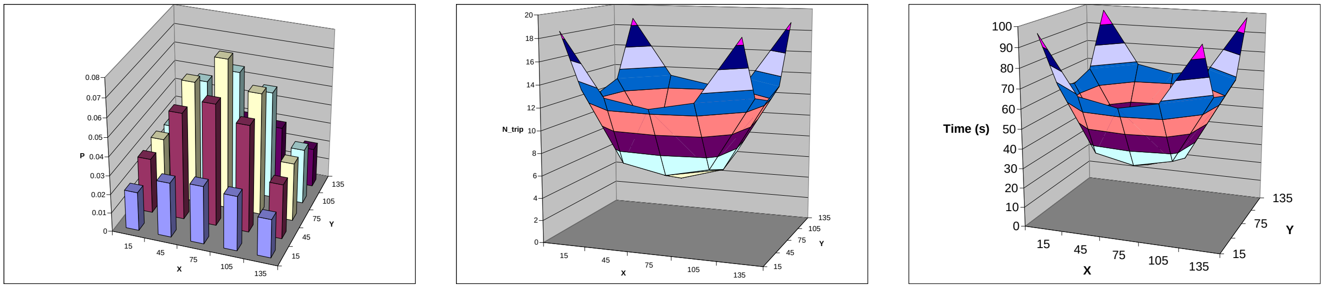
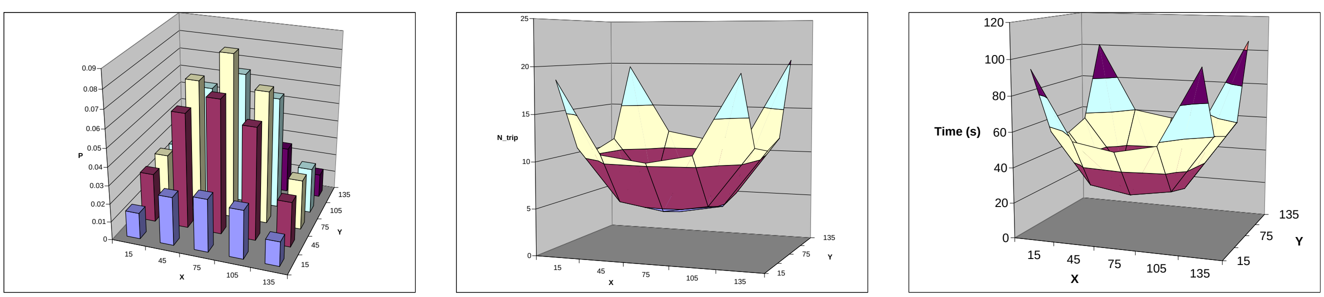


(a) Probability of sensor covering different cells (average = 0.04, maximum = 0.0788)

(b) Expected number of trips before covering a cell (average = 11.431, maximum = 18.667)

(c) Expected time before covering a cell (average = 59.604 s, maximum = 97.353 s)

Fig. 5. Calculated coverage results.



(a) Probability of sensor covering different cells (average = 0.04, maximum = 0.0891)

(b) Average number of trips before covering a cell (average = 10.301, maximum = 20.482)

(c) Average time before covering a cell (average = 52.721 s, maximum = 105.169 s)

Fig. 6. Measured coverage results.